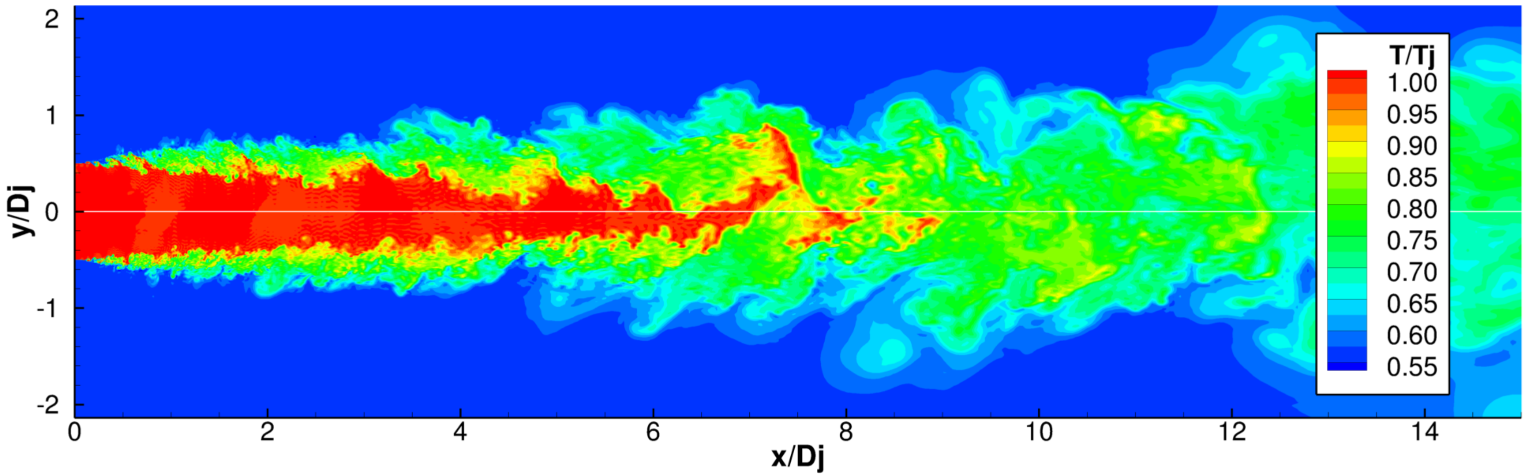
Prediction of Turbulent Temperature Fluctuations in Hot Jets



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Prediction of Heated Jets

- Jets with significant temperature differences have many important applications
  - Aeroacoustics
  - Cooling flows
  - Fuel injectors
  - IR signatures
- Standard CFD methods (RANS) do a very poor job predicting these flows
- Possible reasons
  - Turbulence model
  - Turbulent Prandtl number variation
  - Turbulent heat flux modeling

Reynolds-Averaged Navier-Stokes

$$\frac{\partial \overline{\rho}}{\partial t} + \frac{\partial}{\partial x_i}\left(\overline{\rho}\hat{u}_i\right) = 0$$

$$\frac{\partial}{\partial t}\left(\overline{\rho}\hat{u}_i\right) + \frac{\partial}{\partial x_j}\left(\overline{\rho}\hat{u}_i\hat{u}_j\right) + \frac{\partial \overline{p}}{\partial x_i} - \frac{\partial \overline{\tau}_{ij}}{\partial x_j} + \frac{\partial}{\partial x_j}\left(\overline{\rho u_i' u_j'}\right) = 0$$

$$\frac{\partial}{\partial t}\left(\overline{\rho}\hat{e}_t\right) + \frac{\partial}{\partial x_j}\left(\overline{\rho}\hat{u}_j\hat{e}_t + \hat{u}_j\overline{p}\right) - \frac{\partial}{\partial x_j}\left[\hat{u}_i\overline{\tau}_{ij} - \hat{u}_i\left(\overline{\rho u_i' u_j'}\right)\right] + \frac{\partial}{\partial x_j}\left(\overline{q}_j + c_p\overline{\rho u_j' T'}\right) = 0$$

$$\overline{\tau_{ij}} = 2\mu\left[\frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) - \frac{1}{3}\frac{\partial u_k}{\partial x_k}\delta_{ij}\right]$$

$$\overline{q_j} = -c_p\frac{\mu}{Pr}\frac{\partial T}{\partial x_j}$$

Reynolds-Averaged Navier-Stokes

$$\frac{\partial \overline{\rho}}{\partial t} + \frac{\partial}{\partial x_i}\left(\overline{\rho}\hat{u}_i\right) = 0$$

Reynolds stress

$$\frac{\partial}{\partial t}\left(\overline{\rho}\hat{u}_i\right) + \frac{\partial}{\partial x_j}\left(\overline{\rho}\hat{u}_i\hat{u}_j\right) + \frac{\partial \overline{p}}{\partial x_i} - \frac{\partial \overline{\tau}_{ij}}{\partial x_j} + \frac{\partial}{\partial x_j}\left(\boxed{\overline{\rho u_i' u_j'}}\right) = 0$$

$$\frac{\partial}{\partial t}\left(\overline{\rho}\hat{e}_t\right) + \frac{\partial}{\partial x_j}\left(\overline{\rho}\hat{u}_j\hat{e}_t + \hat{u}_j\overline{p}\right) - \frac{\partial}{\partial x_j}\left[\hat{u}_i\overline{\tau}_{ij} - \hat{u}_i\left(\boxed{\overline{\rho u_i' u_j'}}\right)\right] + \frac{\partial}{\partial x_j}\left(\overline{q}_j + \boxed{c_p\overline{\rho u_j' T'}}\right) = 0$$

$$\overline{\tau_{ij}} = 2\mu\left[\frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) - \frac{1}{3}\frac{\partial u_k}{\partial x_k}\delta_{ij}\right]$$

Turbulent heat flux

$$\overline{q_j} = -c_p\frac{\mu}{Pr}\frac{\partial T}{\partial x_j}$$

Turbulence Closure

- Reynolds stress
  - Boussinesq approximation

$$-\overline{\rho u_i' u_j'} = \mu_t \left(2\hat{S}_{ij} - \frac{2}{3}\frac{\partial \hat{u}_k}{\partial x_k}\delta_{ij} \right) - \frac{2}{3}\overline{\rho} k \delta_{ij}$$

$$\hat{S}_{ij} = \frac{1}{2}\left(\frac{\partial \hat{u}_i}{\partial x_j} + \frac{\partial \hat{u}_j}{\partial x_i} \right)$$

- Turbulent heat flux
  - Gradient diffusion & Reynolds Analogy

$$c_p \overline{\rho u_j' T'} = -c_p \frac{\mu_t}{Pr_t}\frac{\partial T}{\partial x_j}$$

Approach

- Use large-eddy simulation (LES) to examine the turbulent heat flux vector, q^T_i, and turbulent Prandtl number,Pr_t, in hot jets
- Validate the LES using experimental data
  - PIV data for velocity
  - Rayleigh scattering and Raman spectroscopy for temperature
- Make a leap of faith that if u’ and T’ are validated separately, then u’T’ should not be too bad
- Compare results to Reynolds-Averaged Navier-Stokes (RANS) simulations and evaluate

Large-Eddy Simulations

- WRLES code
  - Explicit high-resolution finite-difference code
  - 11-pt DRP differencing scheme (Bogey & Bailly, 2004) with matching filter
  - 4-stage, 3rd order Runge-Kutta time stepping
  - Hybrid MPI/OpenMP parallelization
- Grid
  - Structured grid
  - 36 million points
  - 912x184x181 points downstream of nozzle exit

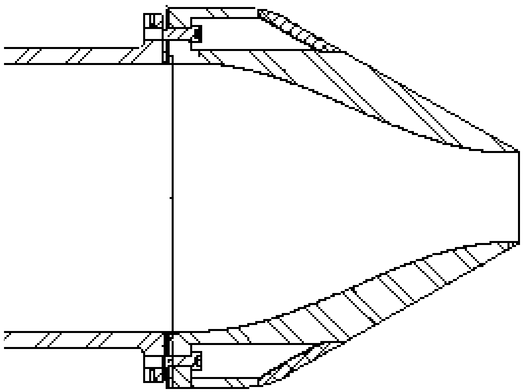
RANS Simulations

- Wind-US
  - Finite-volume
  - Structured grid axisymmetric mode
  - 2nd-order upwind biased RHS
  - Full block-implicit LHS
  - SST-V turbulence model (vorticity based production term)
- Grid
  - Taken from `turbmodels.larc.nasa.gov`
  - 73,151 points
  - Downstream of the nozzle exit 257x251 points
  - Provides grid converged solutions with Wind-US

Round Jet Experiments

- Small Hot Jet Acoustic Rig (SHJAR)
- 2-inch nozzles: ARN2 and SMC000
- PIV Velocity Data
  - Bridges and Wernet, NASA TM 2011- 216807
  - Concensus dataset
  - Verified against hotwire and LDV
- Rayleigh Scattering Temperature Data
  - Mielke et al, AIAA Journal, Vol. 47, No. 4, 2009
  - Point measurement
- Raman Spectroscopy Temperature Data
  - Locke and Wernet, NASA TM 2017-219504
  - Point measurement

ARN2



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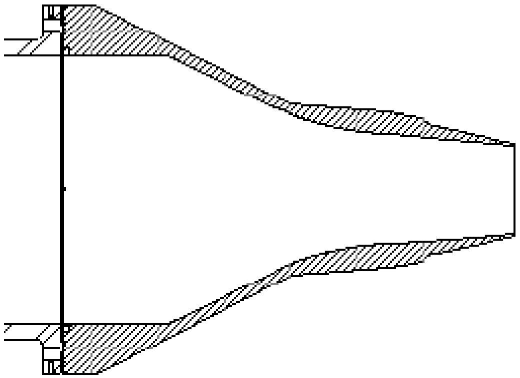


LES Methodology

- Implicit LES
- Nozzle boundary layer
  - No attempt to resolve a turbulent boundary layer
  - Transition occurs quickly in mixing layer
- Non-dimensional time

$$t^* = \frac{tD_j}{U_j}$$

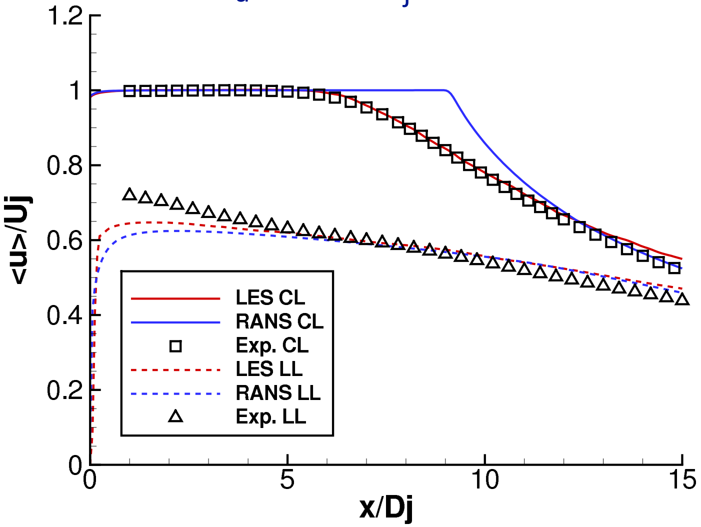
- Startup time: $60t^*$
- Averaging time: > $180t^*$

Flow Conditions

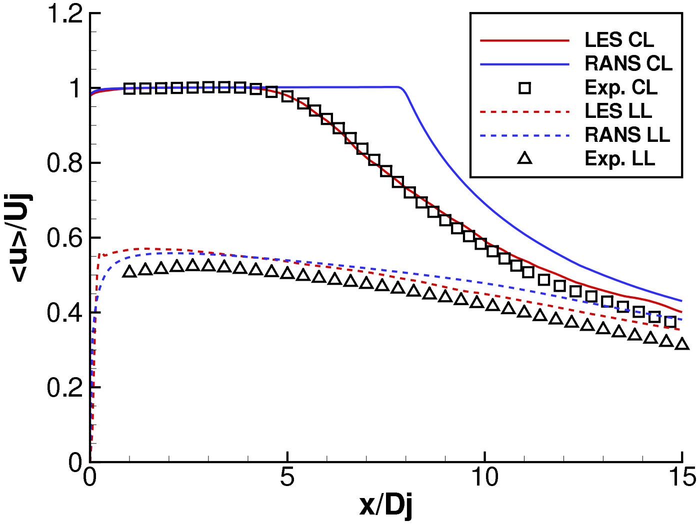
Set Point	M_a	T_j/T_∞	NPR	M_j
3	0.5	0.950	1.197	0.513
23	0.5	1.764	1.102	0.376
27	0.9	1.764	1.357	0.678

Mean Velocity

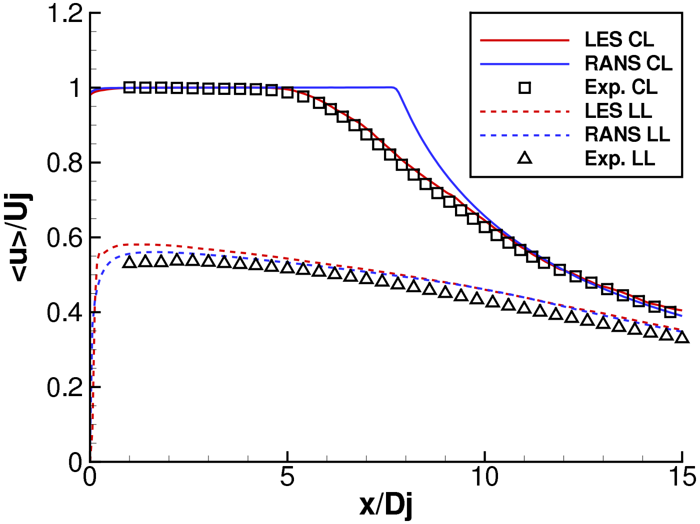
SP 3: $M_a = 0.5$, $T_j/T_\infty = 0.950$



SP 23: $M_a = 0.5$, $T_j/T_\infty = 1.764$

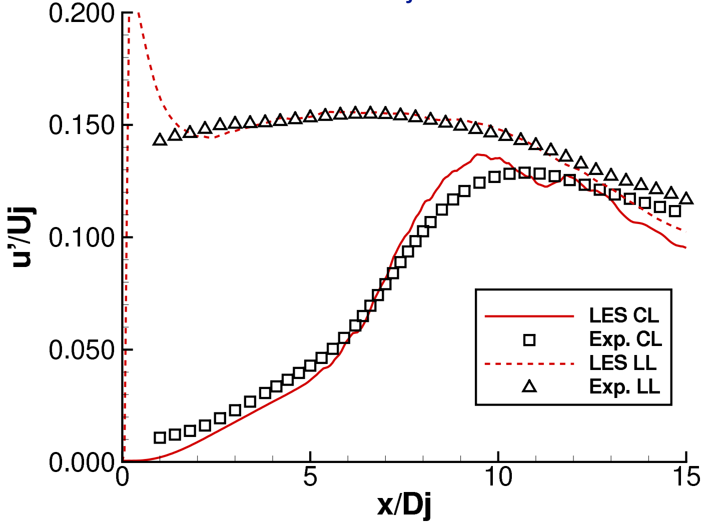


SP 27: $M_a = 0.9$, $T_j/T_\infty = 1.764$

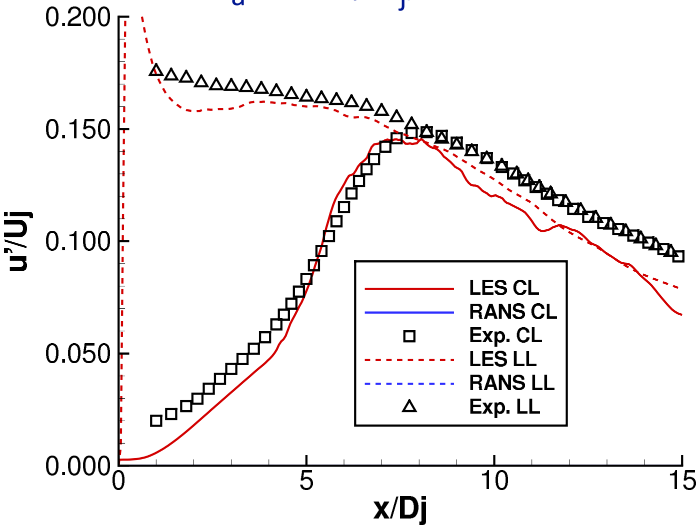


Axial Turbulence Intensity

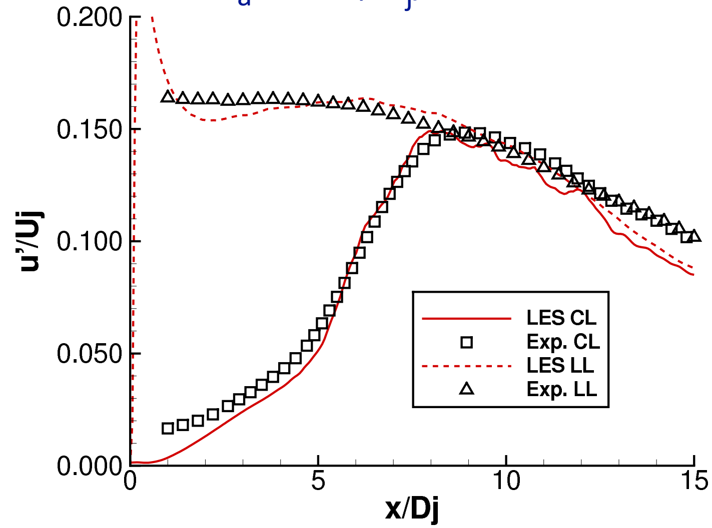
SP 3: $M_a = 0.5$, $T_j/T_\infty = 0.950$



SP 23: $M_a = 0.5$, $T_j/T_\infty = 1.764$

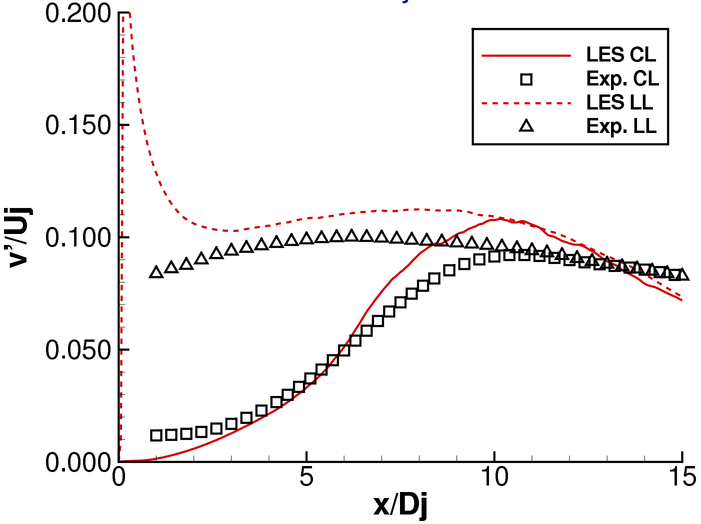


SP 27: $M_a = 0.9$, $T_j/T_\infty = 1.764$

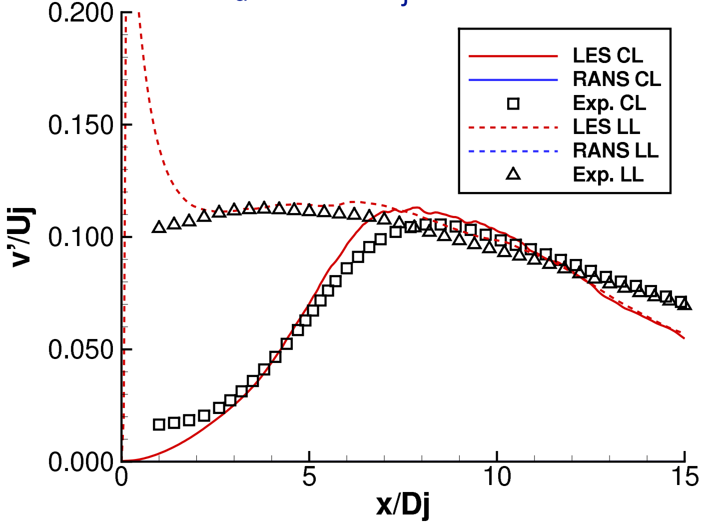


Radial Turbulence Intensity

SP 3: $M_a = 0.5$, $T_j/T_\infty = 0.950$



SP 23: $M_a = 0.5$, $T_j/T_\infty = 1.764$



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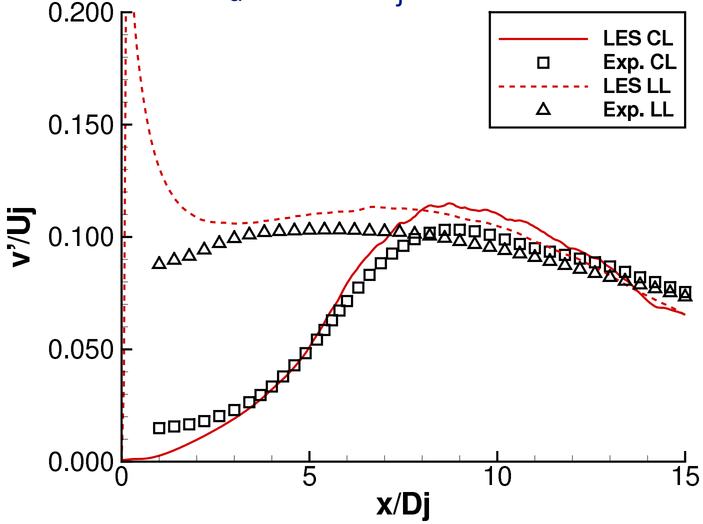
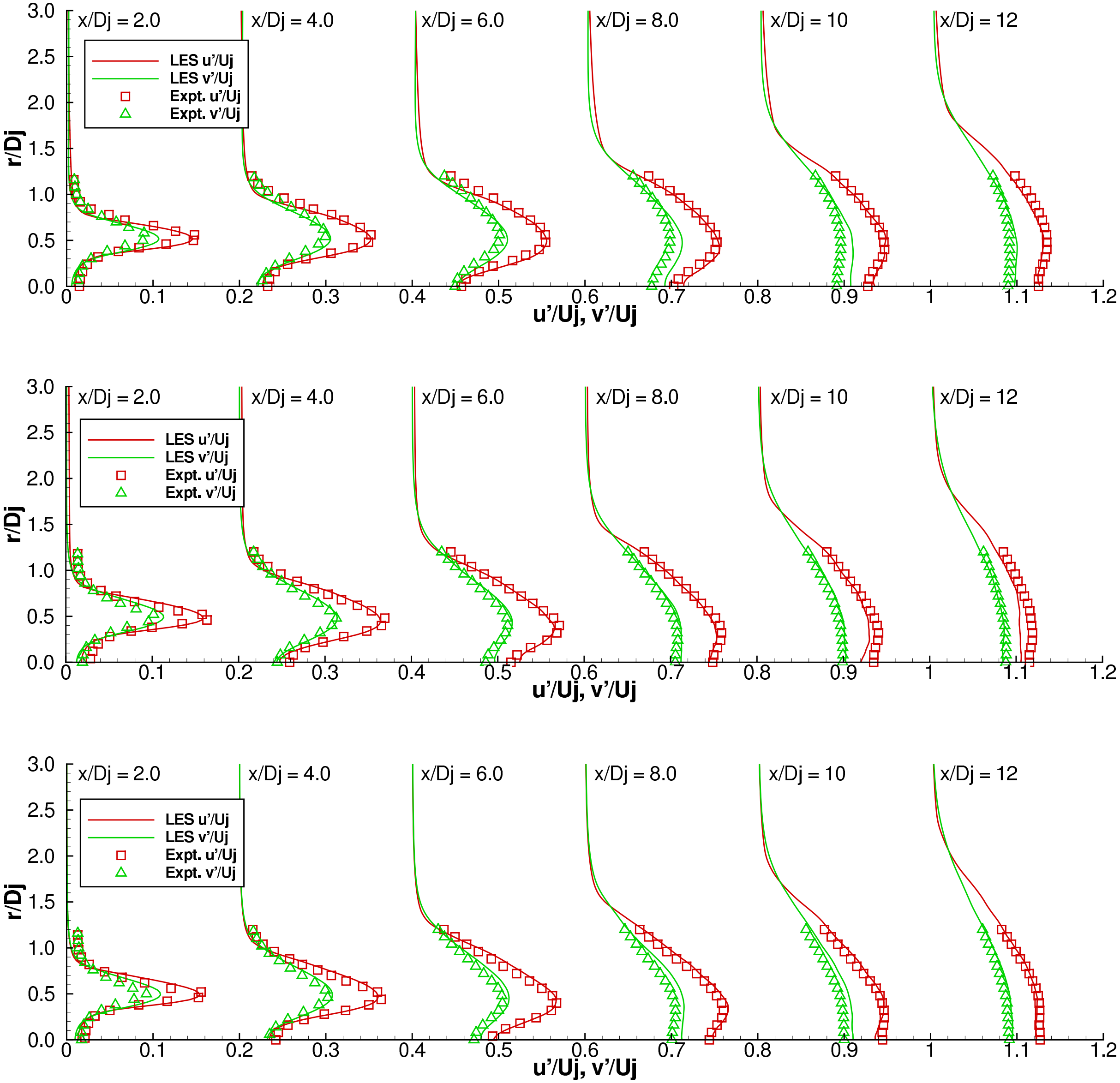


Radial Profiles – u’ & v’

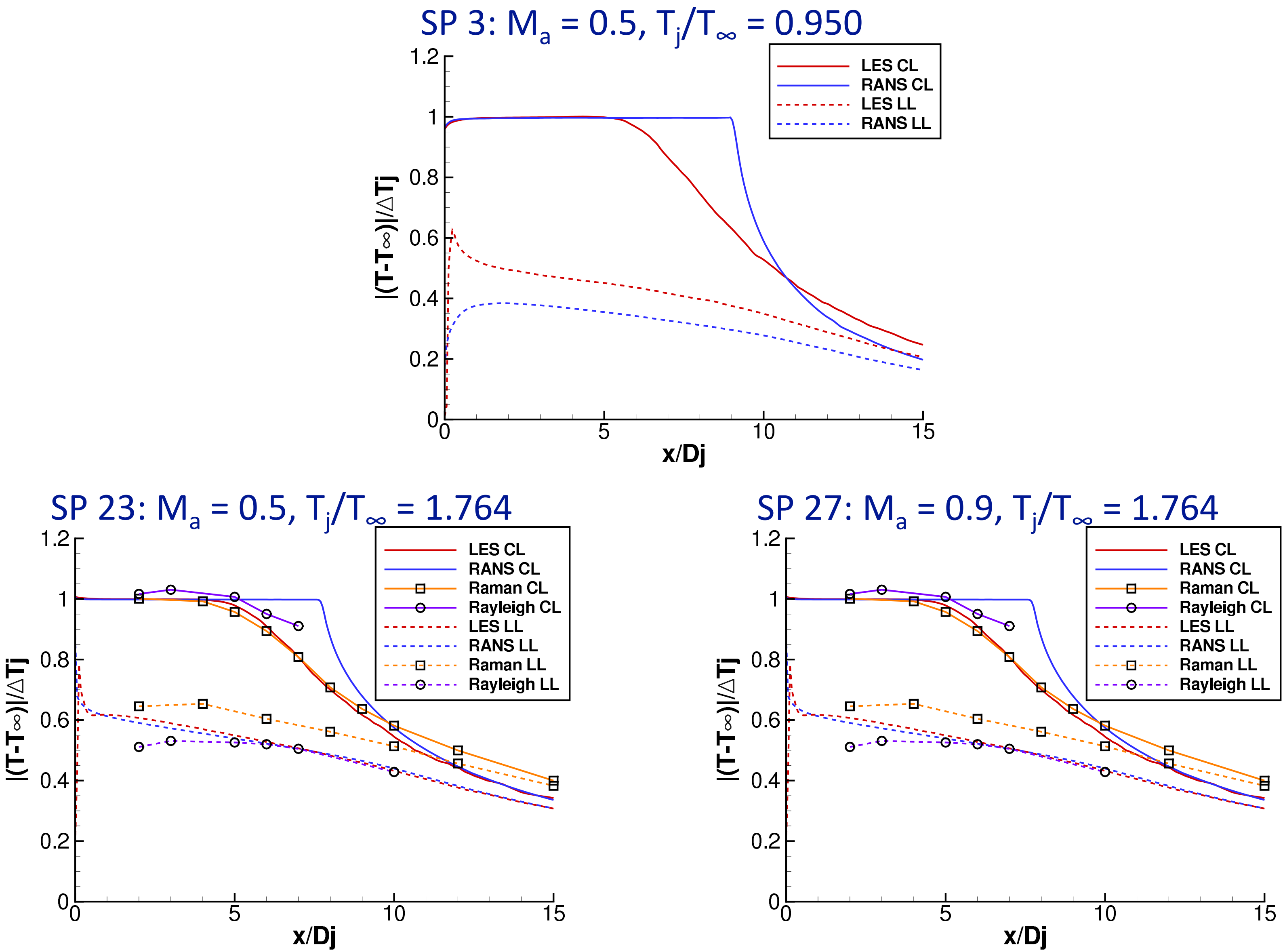
SP 3
$M_a = 0.5$
$T_j/T_\infty = 0.950$

SP 23
$M_a = 0.5$
$T_j/T_\infty = 1.764$

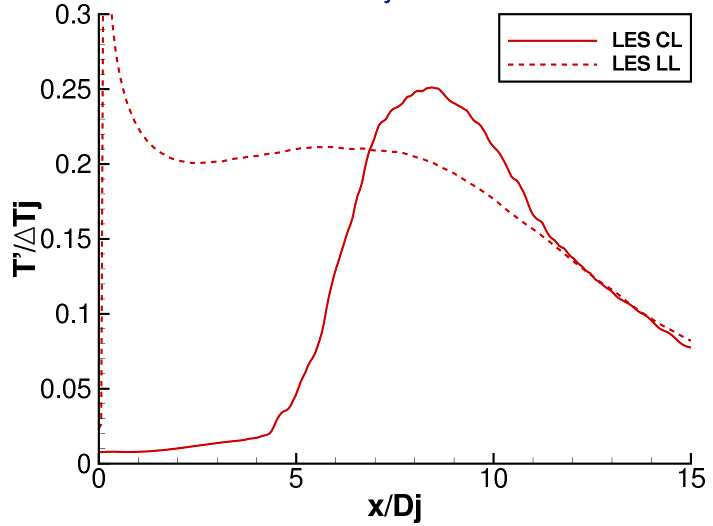
SP 27
$M_a = 0.9$
$T_j/T_\infty = 1.764$

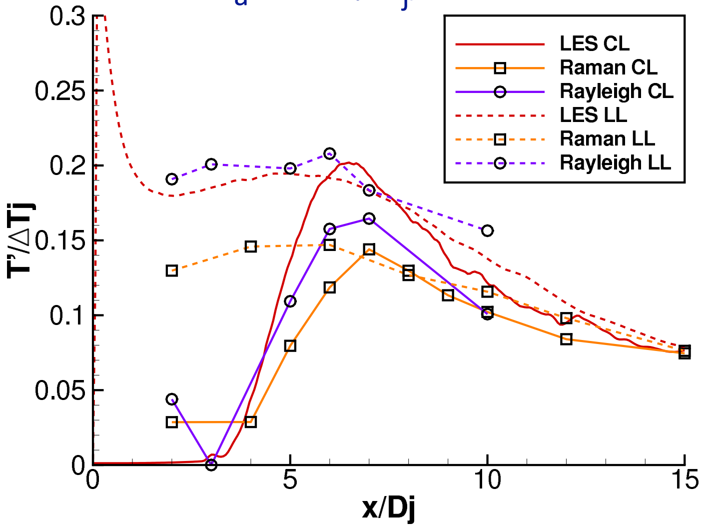


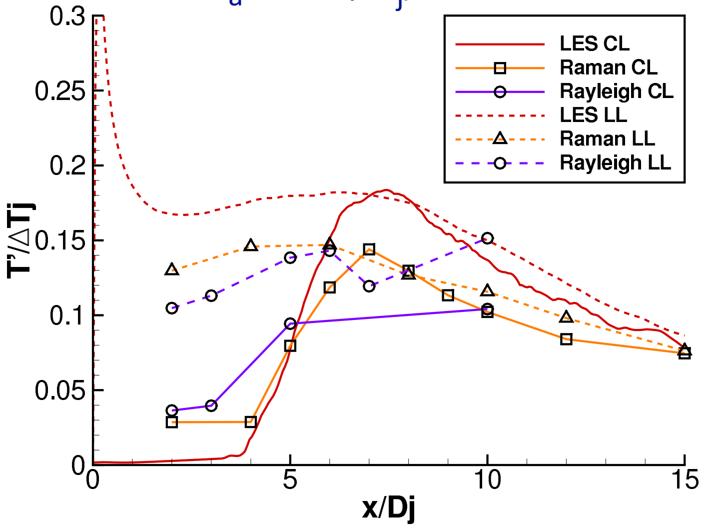
Mean Temperature



RMS Temperature

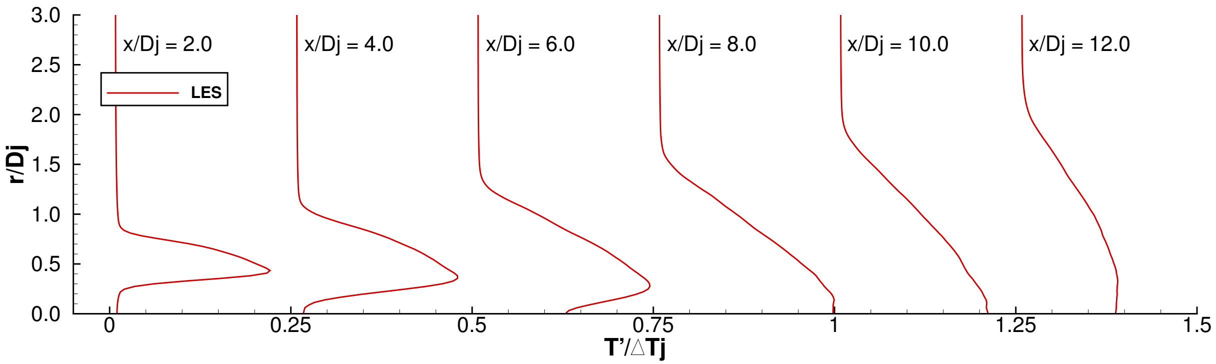




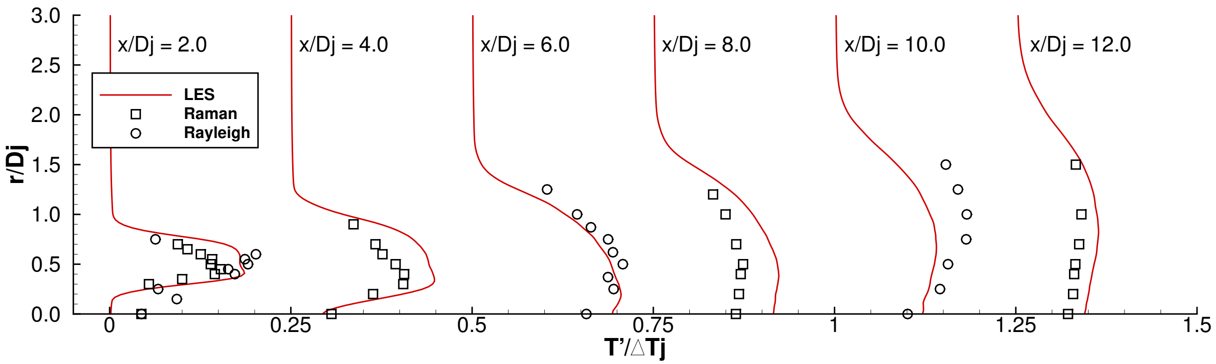


Radial Profiles – T'

SP 3
$M_a = 0.5$
$T_j/T_\infty = 0.950$



SP 23
$M_a = 0.5$
$T_j/T_\infty = 1.764$



SP 27
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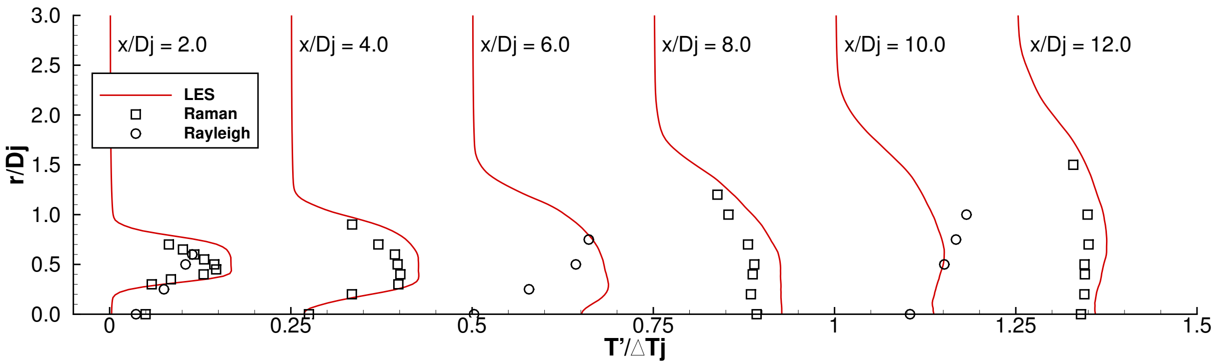
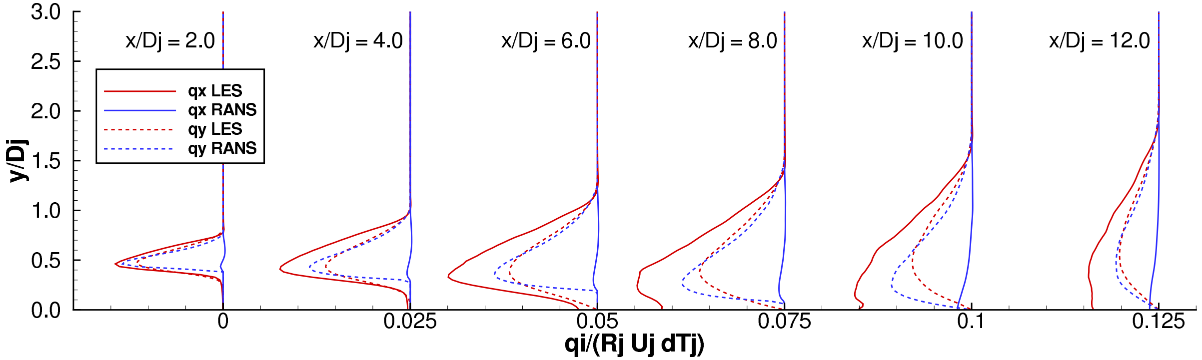
$T_j/T_\infty = 1.764$



Radial Profiles – q^T_x, q^T_y

SP 3
$M_a = 0.5$
$T_j/T_\infty = 0.950$



SP 23
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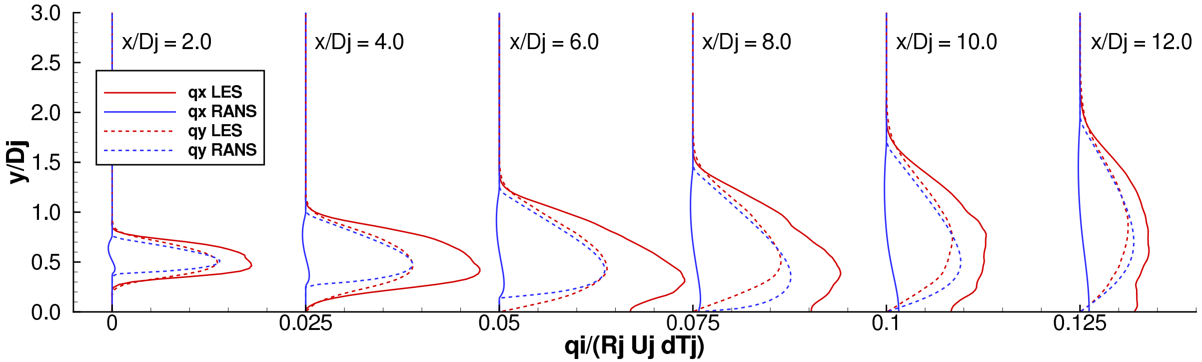
$T_j/T_\infty = 1.764$



SP 27
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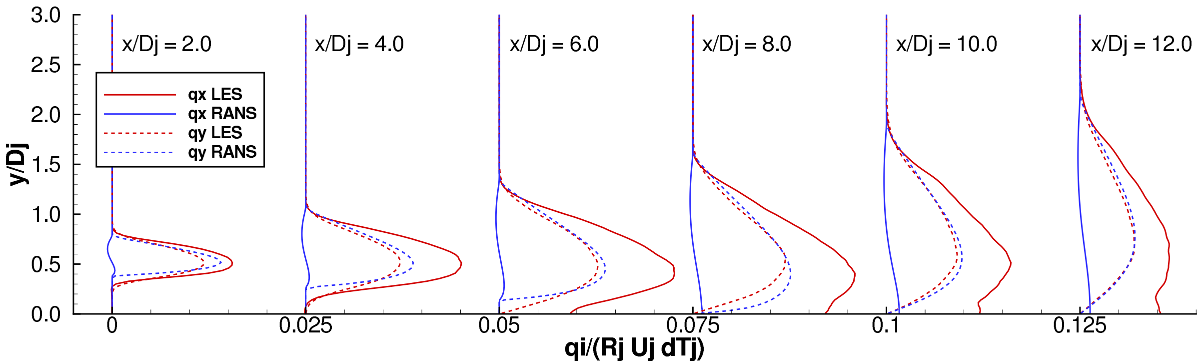
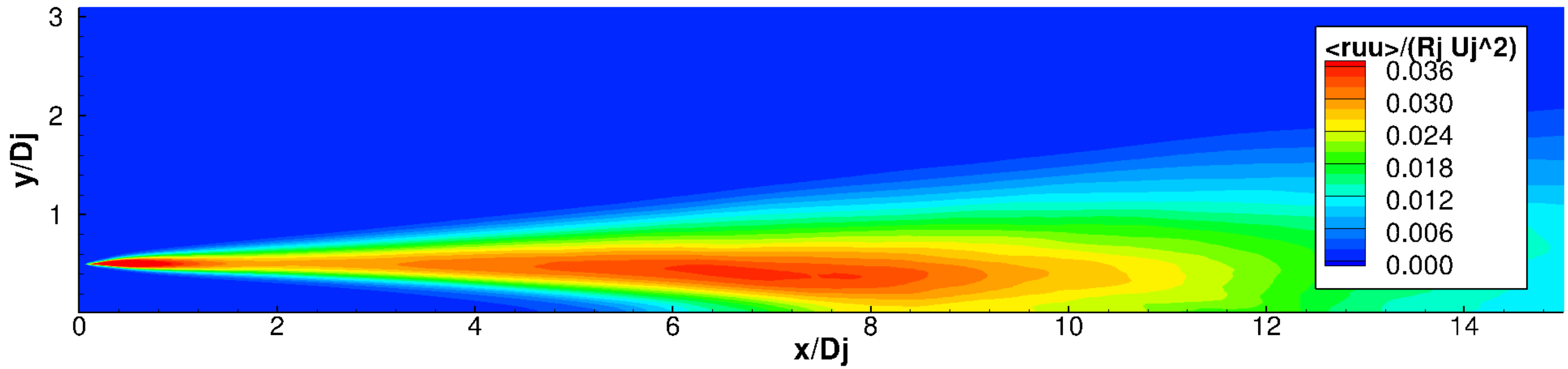
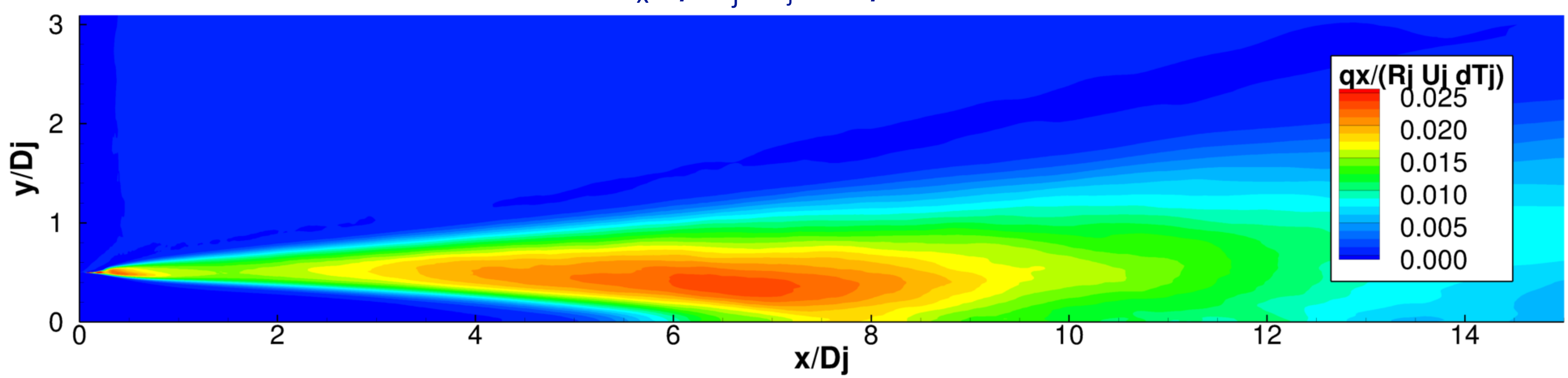


Contours of $<\rho uu>$ and q_x

$<\rho uu>$

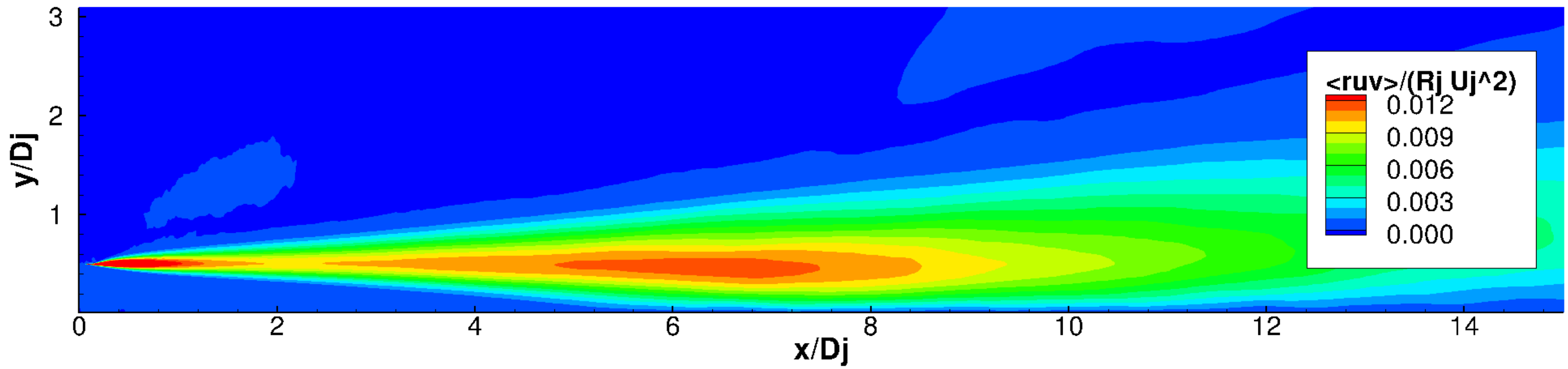


$q_x/(\rho U_j \Delta T_j) = <\rho uT>$

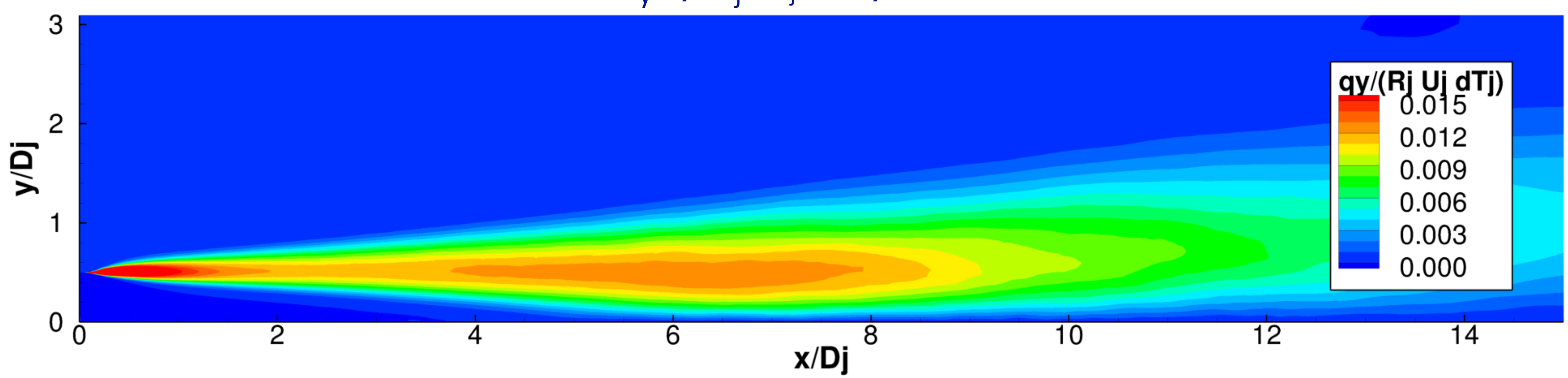


Contours of <ρuv> and q_y

<ρuv>



$q_y/(\rho U_j \Delta T_j)$ = <ρvT>



Turbulent Heat Flux

- Turbulent heat flux model

$$c_p \overline{\rho u_j' T'} = -c_p \frac{\mu_t}{Pr_t} \frac{\partial T}{\partial x_j}$$

- Radial component
  - RANS & LES agree surprising well
  - Mean temperature gradient is in radial direction
- Axial component
  - LES predicts heat flux larger than radial component
  - RANS model predicts almost no heat flux
  - No temperature gradient in this direction
- LES heat flux agrees with experiments in the literature
  - Magnitude
    - Fabris (1979): <uT> & <vT> similar in magnitude
    - Tavoularis & Corrsin (1981): <uT> larger than <vT>
  - Alignment (angle between temp. gradient and heat flux vector)
    - Current LES: 57°
    - Tavoularis & Corrsin (1981): 63°
- Gradient diffusion model is not appropriate for this flow
- Heat flux behavior is analogous to momentum flux

Effect on the Energy Equation

- Energy Equation

$$\frac{\partial}{\partial t}\left(\overline{\rho}\hat{e}_t\right) + \frac{\partial}{\partial x_j}\left(\overline{\rho}\hat{u}_j\hat{e}_t + \hat{u}_j\overline{p}\right) - \frac{\partial}{\partial x_j}\left[\hat{u}_i\overline{\tau}_{ij} - \hat{u}_i\left(\overline{\rho u_i' u_j'}\right)\right] + \boxed{\frac{\partial}{\partial x_j}\left(\overline{q}_j + c_p\overline{\rho u_j' T'}\right)} = 0$$

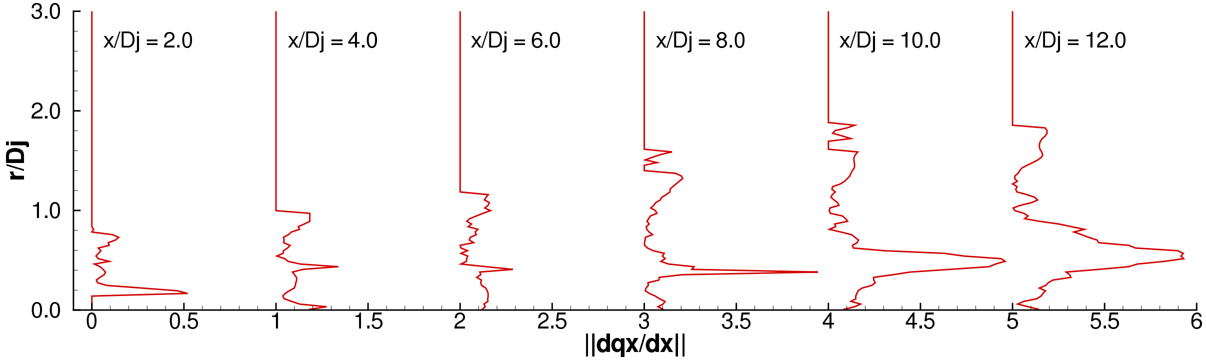
Divergence!

- Quantify the contribution of the missing axial component

$$\left\|\frac{\partial q_x^T}{\partial x}\right\| = \frac{\left|\frac{\partial q_x^T}{\partial x}\right|}{\sqrt{\left(\frac{\partial q_x^T}{\partial x}\right)^2 + \left(\frac{\partial q_r^T}{\partial r}\right)^2}}$$

Radial Profiles – $\|\frac{\partial q_x^T}{\partial x}\|$

SP 3
$M_a = 0.5$
$T_j/T_\infty = 0.950$



SP 23
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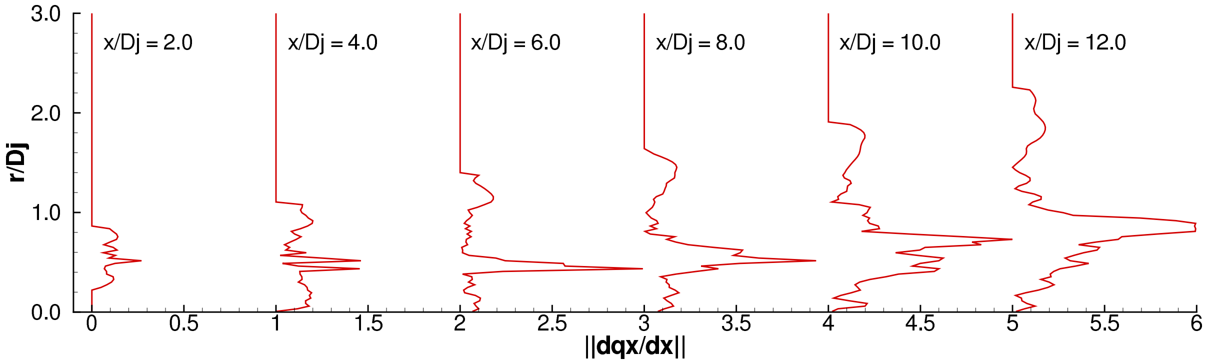

SP 27
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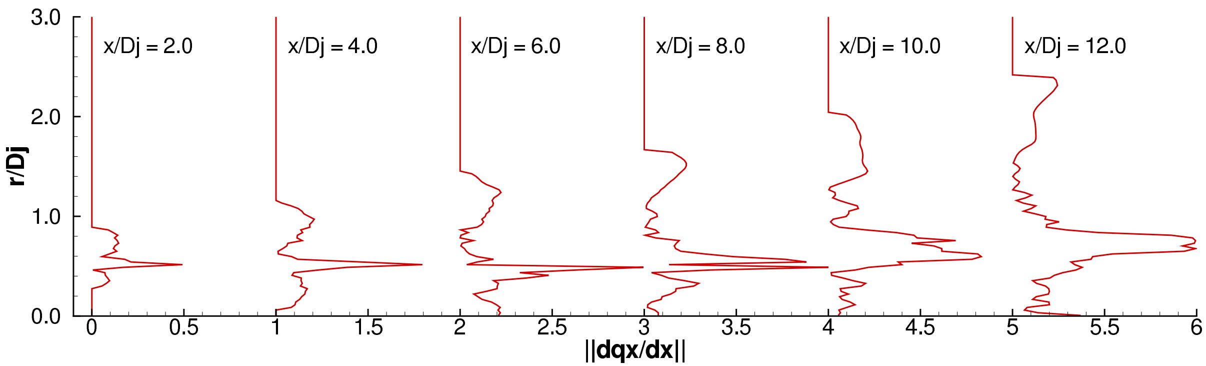
$T_j/T_\infty = 1.764$



Turbulent Prandtl Number

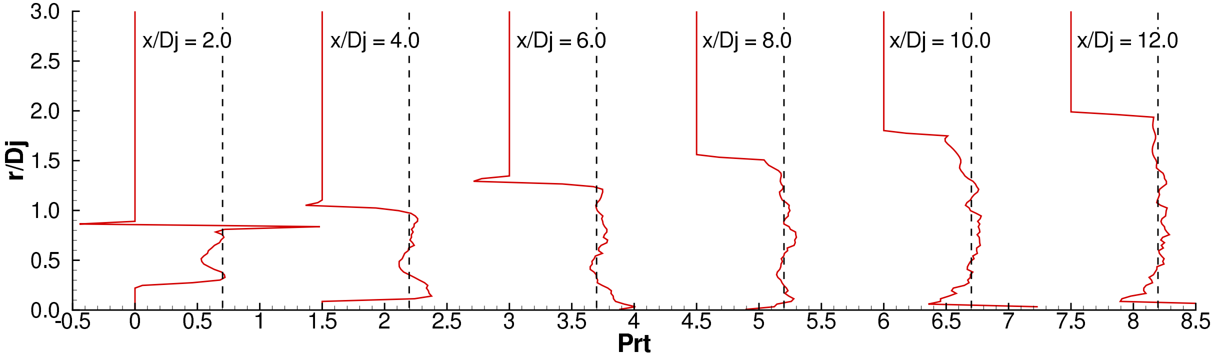
- Treated as a constant but varies, $0.5 < Pr_t < 1.0$
- $Pr_t = 0.7$ is standard value for jets
- Variable Pr_t models often cited as a a solution to these types of problems
- Yoder's (2016) recent results showed no advantage for jets
- Can be computed from the LES

$$\epsilon_m = -\frac{\overline{\rho u' v'}}{\overline{\rho}\frac{\partial \overline{u}}{\partial y}} \qquad \epsilon_T = -\frac{\overline{\rho v' T'}}{\overline{\rho}\frac{\partial \overline{T}}{\partial y}}$$

$$Pr_t = \frac{\epsilon_m}{\epsilon_T}$$

Radial Profiles – Pr_t

SP 3
$M_a = 0.5$
$T_j/T_\infty = 0.950$



SP 23
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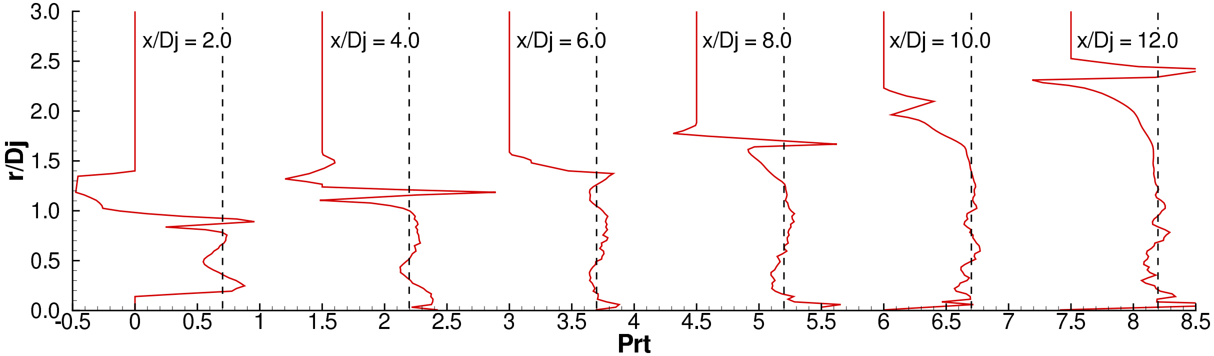

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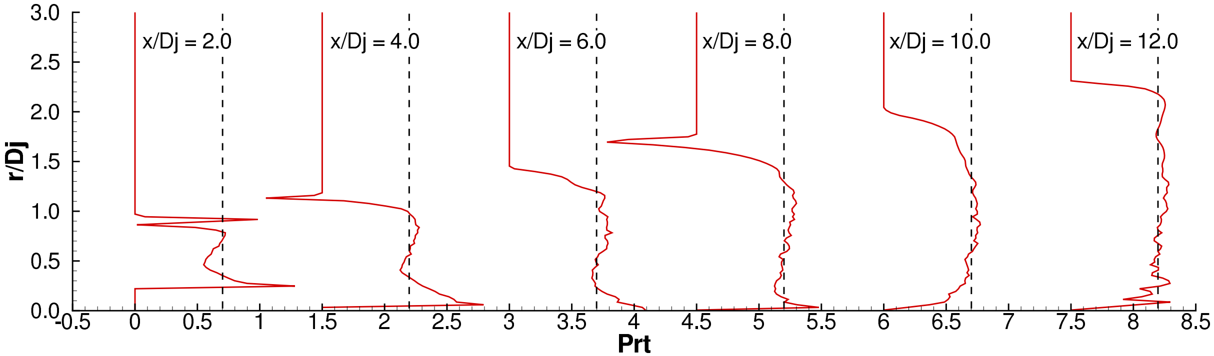

Summary and Conclusion

- LES and RANS methods were used to compute heated jet flows
  - RANS under-predicts spreading rate and inviscid core length (expected result)
  - LES agrees well with experimental data
- Turbulent heat flux
  - LES results consistent with literature
  - RANS model fails to replicate physics
  - Gradient diffusion assumption not appropriate for jets
- Turbulent Prandtl number
  - Little variation within the jet mixing layer
  - $Pr_t = 0.7$ is consistent with literature